NUMBER 128

THE EFFECTS OF THE PRIVATE SECTOR ON THE LABOR BEHAVIOR OF SOVIET COLLECTIVE FARMERS

Clark Chandler

Conference on

STUDIES ON THE SOVIET RURAL ECONOMY

Sponsored by

Kennan Institute for Advanced Russian Studies
The Wilson Center

April 13-14, 1981

1.

THE EFFECTS OF THE PRIVATE SECTOR ON THE LABOR BEHAVIOR OF SOVIET COLLECTIVE FARMERS[1]

1. INTRODUCTION

Labor is an important input into Soviet agricultural production, and its availability has a great impact upon the performance of the Soviet agricultural sector. Therefore, both Soviet and Western scholars have been concerned with identifying the factors affecting the availability of labor to both Soviet agriculture as a whole and the socialized sector of Soviet agriculture in particular.

The existence of the private sector is one of the factors that affects the supply of labor to Soviet agriculture. Soviet collective farms are divided into collective and private sectors. As a result of this division, collective farmers must decide not only how to allocate their time between labor and leisure, but also how to distribute their labor time between the collective and private sectors. This study attempts to identify the most important factors affecting these two types of labor decisions.

2. THEORETICAL MODEL

The Soviet collective farmer's utility is assumed to depend upon income and leisure, and therefore his utility function may

be described as:

$$(1) \qquad U_i = U_i(W_i, h_i^l)$$

where U_i, W_i, and h_i^l represent the utility, income and leisure time of the i-th member of the collective farm.[2]

Leisure is simply the difference between total available time (h_i) and total labor time (h_i^{c+p}). The latter is the sum of time spent working in the collective sector (h_i^c) and time spent working in the private sector (h_i^p). Hence:

$$(2) \qquad h_i^l = h_i - h_i^c - h_i^p$$
$$= h_i - h_i^{c+p}$$

The collective farmer receives income from three sources: (1) wage income earned for work in the collective sector, (2) income earned for work in the private sector, and (3) unearned income. The last of these categories consists of all of the unearned receipts of the collective farmer, whether they come from the government, the social consumption fund of the collective farm, or any other source.

Total wage income from work in the collective sector is equal to the product of the wage rate (w_i) offered to the i-th

member of the collective farm for his work in the collective sector and the total amount of labor time that he spends in the collective sector. It is therefore equal to $w_i h_i^c$.

Income from the private sector is equal to the net value of privately produced output. Each member of the collective farm is entitled to the use of a small plot of land (m_i). The size of this private plot may vary from one collective farm member to another, but is determined independently of labor effort. The private plot and private labor inputs of the collective farmer are combined to produce a product Y according to the following production function:[3]

$$(3) \qquad Y_i = F_i(m_i, h_i^p)$$

Y is then sold by the collective farmer at an exogeneously determined price of P_y to yield $P_y Y_i$ income from employment in the private sector.

Wage income, income from the private sector, and unearned income (W_i') are now added together to obtain total income:

$$(4) \qquad W_i = w_i h_i^c + P_y F_i(m_i, h_i^p) + W_i'$$

Equations (2) and (4) are substituted into (1) to yield:

$$(5) \qquad U_i = U_i(w_i h_i^c + P_y F_i(m_i, h_i^p) + W_i'; h_i - h_i^c - h_i^p)$$

The collective farmer is assumed to maximize utility. Differentiating (5) with respect to h_i^c and h_i^p yields the following first-order necessary conditions:

$$(6a) \qquad \frac{\partial U_i}{\partial h_i^c} = w_i U_i^W - U_i^l \leq 0$$

and

$$\left(\frac{\partial U_i}{\partial h_i^c}\right) h_i^c = 0 \ )$$

$$(6b) \qquad \frac{\partial U_i}{\partial h_i^p} = P_y F_i^h U_i^W - U_i^l \leq 0$$

and

$$\left(\frac{\partial U_i}{\partial h_i^p}\right) h_i^p = 0$$

where:

$$U_i^W = (\partial U_i / \partial W_i), \ U_i^l = (\partial U_i / \partial h_i^l) \text{ and } F_i^h = (\partial F_i / \partial h_i^p).$$

If the collective farmer works in both the collective and private sectors, h_i^c and h_i^p are greater than zero. This implies that the partial derivatives of U_i with respect to these two variables must be strictly equal to zero. Under these circumstances:

$$(7a) \qquad w_i = P_y F_i^h$$

(7b) $$w_i = (U_i^l / U_i^W)$$

and: (1) the socialized sector wage rate is equal to the marginal revenue product (MRP) of labor in the private sector and (2) both are equal to the marginal rate of substitution between income and leisure.

The effects of changes in the exogeneous variables of the model (w_i, W_i', m_i, and P_y) on total labor time can be determined by totally differentiating (7b) with respect to these variables. Equation (7a) is then differentiated with respect to these variables to determine their impact upon private labor time. This is done in Appendix A, and the results are reported in Table 1. Changes in the amount of labor time spent in the collective sector are equal to the difference between changes in total labor time and changes in private labor time ($dh_i^c = dh_i^{c+p} - dh_i^p$).

Changes in the collective sector wage rate have an ambiguous effect on both total and collective labor time due to the familiar countervailing pull of the income and substitution effects. But an increase in collective sector wage rates necessarily reduces private labor time as the collective farmer transfers labor from private to collective employment until the MRP of labor in the private sector increases until it equals the new wage rate.

An increase in unearned income unambiguously decreases

Table 1. Predicted Labor Behavior of Collective Farmers (No Compulsory Minimum Labor Constraints)

	dw_i	dW'_i	dm_i	dP_y
dh_i^{c+p}	?	-	-	-
dh_i^{c}	?	-	-	-
dh_i^{p}	-	0	+	+

total labor time as income increases while wages do not change. This entire decrease in labor time takes place in the collective sector as private labor inputs must remain at their initial level to preserve the equality between the collective sector wage rate and the MRP of labor in the private sector.

Changes in m_i and P_y have qualitatively equivalent effects on labor behavior. Increases in either of these two variables lead to an increase in income without changing the marginal wage rate (which is pegged at the wage rate offered by the collective sector), and therefore reduce total labor time. The reduction in collective labor time is greater than the reduction in total labor time, as labor is transfered from the collective to the private sector as a result of the upward shift in the private MRP of labor curve.

The above model describes the collective farmer's labor behavior if he is free to make his labor decisions according to his own wishes. This has frequently not been the case, however, due to the existence of compulsory labor minimums that require each collective farmer to spend a specified minimum amount of time working in the collective sector. These compulsory minimums are now introduced into the model by specifying that each collective farmer must spend at least $h_i^{c'}$ amount of time working in the collective sector. With the introduction of this constraint, equation (5) can be re-written as:

$$(5^*) \qquad U_i = U_i(w_i(h_i^{c'} + h_i^{c*}) + P_y F_i(m_i, h_i^p) + W_i'; h_i - h_i^{c'} - h_i^{c*} - h_i^p)$$

where $h_i^{c*} = h_i^c - h_i^{c'}$, and denotes voluntary labor time spent in the collective sector (i.e., labor time in excess of the required minimum). Differentiating (5*) with respect to h_i^{c*} leaves (6b) unchanged, but (6a) becomes:

$$(6a^*) \qquad \frac{\partial U_i}{\partial h_i^{c*}} = w_i U_i^W - U_i^l \leq 0$$

and

$$\left(\frac{\partial U_i}{\partial h_i^{c*}}\right) h_i^{c*} = 0$$

The minimum labor constraint $h_i^{c'}$ is binding if it forces the collective farmer to work more in the collective sector than he would chose to voluntarily. This occurs when the MRP of labor in the private sector is greater than the wage rate offered by the collective sector. $(\partial U_i/\partial h_i^{c*})$ is strictly less than zero under these circumstances, and therefore $h_i^{c*} = 0$ and $h_i^c = h_i^{c'}$.

Using equation (6b):

$$(8) \qquad P_y F_i^h = (U_i^l/U_i^W)$$

and the marginal rate of substitution between leisure and income is given by the MRP of labor in the private sector. Labor behavior under these circumstances can be analyzed by totally differentiating (8) with respect to the exogeneous variables of the model. Noting that changes in total labor time are the same as changes in private labor time, the results of this differentiation (done in Appendix A) are reported in Table 2.

Increases in the collective sector wage rate and in unearned income reduce both total and private labor time due to the income effect. This result for wage changes comes about because changes in the collective sector wage rate change income, but do not have a direct effect[6] on the cost of leisure (now given by the MRP of labor in the private sector).

Table 2 Predicted Labor Behavior of Collective Farmers (With Compulsory Minimum Labor Constraints)

	dw_i	dW_i'	dm_i	dP_y	$dh_i^{c'}$
dh_i^{c+p}	-	-	?	?	+
dh_i^{c}	0	0	0	0	1
dh_i^{p}	-	-	?	?	-

Changes in plot size and in the price of private output now have an ambiguous effect on labor behavior. An increase in either of these variables leads to an increase in both income and the marginal cost of leisure. The income and substitution effects therefore work in opposite directions.

Finally, an exogenous increase in the minimum labor constraint leads to an increase in total labor time and a decrease in private labor time (with the amount of labor time spent in the collective sector changing by the same amount as the change in the constraint). This result is not surprising. If private labor time were to increase following an increase in the minimum labor constraint, the collective farmer's total income would increase and the MRP of labor in the private sector would fall. Both of these effects would tend to decrease total labor time, which is incompatible with a simultaneous increase in both col-

lective and private labor time. On the other hand, if total labor time falls following an increase in the minimum labor constraint, total income would fall (as $dW_i = w_i dh_i^{c'} - P_y G_i^h dh_i^p = 0$ under these circumstances) and the MRP of labor in the private sector would increase (as private labor inputs would necessarily decrease). This once again leads to a contradictory result as both of these effects would push for an increase in total labor time. Therefore the only feasible result is a decrease in private labor time that is less than the forced increase in collective labor time.

The comparative static results point out the basic incompatibility between economic incentives and institutional constraints on labor behavior. If compulsory minimum constraints are used to secure labor inputs into the collective sector, then the only way of either increasing or decreasing labor inputs into this sector is through the increase or decrease of the statutory minima. The use of other tools has no effect on labor inputs into the socialized sector and may have perverse effects on total and private labor time.

3. ESTIMATION OF LABOR SUPPLY FUNCTIONS

The total, collective and private labor supply functions of collective farmers can be derived from equations (7a) and (7b). Linear approximations of these equations are then obtained through a first-order Taylor expansion to yield:

$$(9a) \qquad h_i^c = a_0 + a_1 w_i + a_2 m_i + a_3 W_i' + e_i^c$$

$$(9b) \qquad h_i^p = b_0 + b_1 w_i + b_2 m_i + e_i^p$$

The individual data needed to estimate these labor supply functions are not available. Therefore, grouped data are used, and each observation denotes the mean value of the variables in a particular republic in a given year. The collective sector labor supply function was estimated using data for the fifteen Soviet republics in the years 1963, 1967 and 1970; and the private sector labor supply function was estimated using data for the fifteen republics in 1968. Separate labor supply functions were estimated for able-bodied men and women.

The labor supply functions presented in (9a) and (9b) assume that the labor behavior of collective farmers is not restricted by compulsory minimum labor constraints. If such binding constraints exist, then collective labor inputs are determined

exogeneously and the coefficients in (9a) should not be significantly different from zero. Therefore the presence of binding minimum labor constraints may bias the coefficient estimates, with the degree of bias depending upon the number of farmers affected by these constraints.

Collective farmers received much lower wages for their work in the collective sector in 1963 than in the second half of the 1960s. Therefore it is possible that while compulsory minimum labor constraints were needed to force collective farmers to work in the collective sector in 1963, wage incentives alone were sufficient in the latter two years.[5] This possibility is acknowledged by introducing a dummy variable into (9a) that allows the coefficient values in 1963 to vary from those in 1967/1970. The model to be estimated becomes:

$$(10a) \qquad h^c_{gt} = a_0 + a_1 w_{gt} + a_2 m_{gt} + a_3 W'_{gt} + a_4 Z_t + a_5 Z_t w_{gt} + a_6 Z_t m_{gt} + a_7 Z_t W'_{gt} + e^c_{gt}$$

$$(10b) \qquad h^p_g = b_0 + b_1 w_g + e_2 m_g$$

where the subscripts g and t denote the g-th republic and the t-th year respectively, and Z_t equals 1 in 1963 and 0 in 1967 and 1970.

In these equations, labor time is measured in man-days per

year, wages in rubles per man-day (including both monetary payments and payments-in-kind), plot size in hectares per collective farmer, and unearned income in rubles per year. The values of the independent variables denote the average for all able-bodied collective farmers, and are therefore assumed to be the same for both able-bodied men and women.[6] The error terms are assumed to be heteroskedastic across specified groups of republics, and to follow a first-order autocorrelation scheme over time. The appropriate corrections were made for these problems.

The estimation results are presented in Table 3. The values of the standard errors are reported in parenthesis below the coefficient estimates. One, two, and three asterisks indicate that the coefficients are signficantly different from zero at the 90, 95, and 99 percent level respectively. The R^2s reported in the table refer to the transformed equations, and therefore cannot be interpreted as a measure of the share of the labor behavior of collective farmers that is explained by the independent variables.

The estimation of the labor supply functions of able-bodied men provided no surprises. The collective sector wage coefficient and the private sector plot coefficient are both positive and significantly different from zero at the 95 and 90 percent level respectively. None of the other coefficients are significantly different from zero. This may either be due to the large amount of multicollinearity that exists among the explanatory variables,

which weakens the power of the econometric tests, or it may imply that the coefficients are in fact equal to zero. The insignificance of the collective sector plot and the private sector wage coefficients may imply that the two sectors do not compete for labor - able-bodied male collective farmers work full-time in the collective sector and only work on their private plots when such work does not interfere with their labor in the collective sector (i.e., in the evenings, on their days off, etc.). And the income coefficient may be equal to zero for either of two reasons. First, the income data used were based on the Soviet definition of income, and the largest part of unearned income consists of free services (medical, educational, recreational, etc.). It is not very surprising that income of this type does not have a strong impact on the labor behavior of able-bodied men. Second, the income levels of collective farmers may be low enough so that income is valued so highly in comparison with leisure that the income effect is negligible.

The fact that none of the dummy variables that have been included in the model to measure the marginal effect of being in 1963 as opposed to the two later years can be shown to be significantly different from zero implies that the labor supply functions of able-bodied men were the same in 1963 as in 1967/1970. To corroborate this, an F-test was used to test for the joint effect

of the four dummy variables. Their joint effect could not be shown to be significantly different from zero at the 95 percent level. These results suggest that economic factors rather than compulsory minimum labor constraints were the primary determinants of the labor behavior of able-bodied male collective farmers as early as 1963.

As in the case of able-bodied men, the estimated collective sector wage coefficient and private sector plot coefficient were both positive and significantly different from zero for able-bodied women, this time at the 90 and 99 percent level respectively. Moreover, the estimated income and plot coefficients of the collective sector labor supply function of able-bodied women could not be shown to be significantly different from zero, and therefore there is no indication that these variables affect the labor behavior of able-bodied women in the collective sector.

But there are also a number of surprises in the estimation results for able-bodied women. The first of these is that although the predicted wage coefficient of the private sector labor supply function is negative, the estimated coefficient for able-bodied women is both positive and significantly different from zero at the 95 percent level. There are two likely causes of this perverse result. One, women may specialize more in the private sector as wage levels increase. Higher wage rates may increase the comparative advantage of men in the collective

sector (as men are more likely to have the skills most needed in the collective sector than women). As a consequence of this, women may be displaced by men in the collective sector and therefore increase the amount of time they spend working in the private sector. A second and complementary explanation is that the private sector production function has been misspecified in the theoretical model. Land and labor are the only inputs in this production function. However, livestock is one of the most important products of the private sector, and the production of livestock requires fodder, food-grains, etc. which can only be obtained from the socialized sector. If an increase in the collective sector wage rates leads to an associated increase in the ability of collective farmers to acquire such inputs (perhaps via payments in kind), then wage increases may lead to increases in both private sector output and associated labor inputs.

The coefficients of three of the four dummy variables included to measure the marginal effects of being in 1963 are also significantly different from zero, indicating that the labor behavior of able-bodied women changed between 1963 and 1967/1970. The 1963 collective sector labor supply function for able-bodied women is therefore presented in Table 4 to illustrate the nature of these changes.

The observed changes do not appear to be consistent with the

hypothesis that minimum labor constraints have become less important in recent years, however. For although the change in the wage coefficient is consistent with this hypothesis, the change in the plot coefficient is not.

Therefore it seems likely that the differences between the estimated labor coefficients in the early and late 1960s are due to changes in the composition of the female labor force. Soviet writers frequently point out that collective farm women can be divided into two different groups of workers: one group consisting of women who work in highly seasonal occupations (such as unskilled field hands) and the other of women who work in occupations which demand long hours consistently throughout the year (such as milk-maids who must be available for morning and evening milkings day after day). Women who fall into the first of these two categories have time to work in the private sector, and consequently their labor behavior is likely to be sensitive to changes in private plot size. Moreover, as these women are likely to have children and therefore be unwilling to work more than a specified amount of time in the collective sector, and as their jobs are highly seasonal and therefore the demand for their labor is likely to be highly inelastic, the labor behavior of these women may not be very sensitive to changes in wage rates. Women belonging to the second of the above groups, on the other hand, do not have time to work in

the private sector and therefore their labor behavior should not be affected by plot size but is likely to be affected by wage rates. Given the existence of these two groups, one explanation for the observed changes in the wage and plot coefficients is that the 1960s saw a relative decline in the importance of women who divided their labor time between the private and collective sectors and a corresponding increase in the importance of women who worked exclusively in the collective sector.

4. SUMMARY

The model that has been developed and tested in this paper is subject to a number of limitations. The model itself is a very simple one, and ignores such problems as wage determination, the use of one sector's output as input in the other sector, and the fact that the key decision-making unit on the collective farm may be the household rather than the individual worker. Moreover, the use of highly aggregated data has necessitated, at least in the case of able-bodied women, grouping together different types of women who may have different labor supply functions. And finally, regional differences in prices, land quality, etc. have been ignored - something that cannot be done with much comfort in the case of a country as large and as diverse as the Soviet Union.

Given these limitation, the study does indicate that the

labor behavior of Soviet collective farmers is heavily influenced by economic variables. Moreover, private plot size apparently has no effect on the collective labor time of able-bodied men, and in general there is little evidence that the private and collective sectors compete for the same scarce labor resources. Finally, the estimation results suggest that the labor behavior of able-bodied women is both more complicated and more heavily influenced by the private sector than that of able-bodied men.

Footnotes

1. The work presented in this paper was carried out as part of the author's doctoral dissertation at the University of Michigan.

2. This utility function is assumed to have positive first-order derivatives, negative second-order own partial derivatives, and positive second-order cross-partial derivatives.

3. This production function is assumed to have positive marginal products and diminishing returns to both factors.

4. This is not strictly true. Private labor inputs decrease as a result of the increase in income, and therefore the MRP of labor in the private sector and the cost of leisure both increase. But since this effect is induced by the initial increase in income, there is no chance that it will outweigh the income effect.

5. Several different methods of allowing for the possibility of minimum labor constraints were considered. One was to categorize the dummy variable by wage levels. There are two problems with this approach: (1) it is very difficult to select the appropriate wage levels and (2) wage levels may vary from one group to another. Alternatively, the assumption could have been made that the share of collective farm members bound by the minimum labor constraint was a continuous function of the wage rate during the transition period. This approach envisions a continuous movement of collective farm members from a situation in which their labor behavior is governed by legal restrictions to one in which it is determined by economic factors rather than the more abrupt transition implied by the use of dummy variables. This approach would require both a specific modelling of the transition function and some knowledge of the beginning and end points of the transition period.

6. This makes it very difficult to compare the wage coefficients of able-bodied men and women, as these coefficients may differ due to different skill coefficients as well as (possibly) different income and substitution effects.

APPENDIX A

THE DERIVATION OF THE COMPARATIVE STATIC RESULTS

To derive (3.10a), (3.9) is totally differentiated with respect to α. This yields:

$$\frac{dw_i}{d\alpha} = \frac{U_i^W[U_i{}^{W^1}\frac{dW_i}{d\alpha} + U_i^{11}\frac{dh_i^1}{d\alpha}] - U_i^1[U_i^{WW}\frac{dW_i}{d\alpha} + U_i^{W^1}\frac{dh_i^1}{d\alpha}]}{(U_i^W)^2}$$

$$= \frac{[U_i^W U_i{}^{W^1} - U_i^1 U_i^{WW}]}{(U_i^W)^2}\frac{dW_i}{d\alpha} + \frac{[U_i^W U_i^{11} - U_i^1 U_i^{W^1}]}{(U_i^W)^2}\frac{dh_i^1}{d\alpha}$$

Solving this expression for $dh_i^1/d\alpha$ yields:

$$\frac{dh_i^1}{d\alpha} = -\frac{dh_i^{c+p}}{d\alpha} = -\frac{A}{B}\frac{dW_i}{d\alpha} + \frac{1}{B}\frac{dw_i}{d\alpha}$$

where:

$$A = \frac{[U_i^W U_i^{1W} - U_i^1 U_i^{WW}]}{(U_i^W)^2} \text{ and } B = \frac{[U_i^W U_i^{11} - U_i^1 U_i^{W^1}]}{(U_i^W)^2}$$

If the first-order partial derivates of the utility function are positive, the second-order own partial derivatives

negative, and the second-order cross partial derivatives are positive, then $A > 0$ and $B < 0$, and this yields the signs given in the parenthesis of equation (3.10a).

The sign of $dh_i^1/d\alpha$ can now be determined if the signs for $dW_i^1/d\alpha$ and $dw_i/d\alpha$ are known. These are:

for $dw_i = d\alpha$:

$$\frac{dW_i}{dw_i} > 0; \quad \frac{dw_i}{dw_i} = 1 > 0 \Rightarrow \frac{dh_i^1}{dw_i} \text{ is } \lesseqgtr 0$$

for $dW_i' = d\alpha$

$$\frac{dW_i}{dW_i'} > 0; \quad \frac{dw_i}{dW_i'} = 0 \Rightarrow \frac{dh_i^1}{dW_i'} > 0$$

for $dm_i = d\alpha$

$$\frac{dW_i}{dm_i} > 0; \quad \frac{dw_i}{dm_i} = 0 \Rightarrow \frac{dh_i^1}{dm_i} > 0$$

for $dP_y = d\alpha$

$$\frac{dW_i'}{dP_y} > 0; \quad \frac{dw_i}{dP_y} = 0 \Rightarrow \frac{dh_i^1}{dP_y} > 0$$

To derive (3.10b), (3.8) is totally differentiated with respect to α. This yields:

$$\frac{dw_i}{d\alpha} = P_y\left[G_i^{hh}\frac{dh_i^P}{d\alpha} + G_i^{hm}\frac{dm_i}{d\alpha}\right] + G_i^h\frac{dP_y}{d\alpha}$$

$$\frac{dh_i^P}{d\alpha} = \frac{(dw_i/d\alpha)}{P_y G_i^{hh}} - \frac{G_i^{hm}(dm_i/d\alpha)}{P_y G_i^{hh}} - \frac{G_i^h(dh_i^P/d\alpha)}{P_y G_i^{hh}}$$

where G_i^h, $G_i^{hm} > 0$ and $G_i^{hh} < 0$ by assumption. The sign of $dh_i^P/d\alpha$ can now be determined if the signs of $dw_i/d\alpha$, $dm_i/d\alpha$ and $dP_y/d\alpha$ are known.

for $dw_i = d\alpha$:

$$\frac{dw_i}{dw_i} = 1 > 0;\ \frac{dm_i}{dw_i} = \frac{dP_y}{dw_i} = 0 \Rightarrow \frac{dh_i^P}{dw_i} < 0$$

for $dW_i' = d\alpha$

$$\frac{dw_i}{dW_i'} = \frac{dm_i}{dW_i'} = \frac{dP_y}{dW_i'} = 0 \Rightarrow \frac{dh_i^P}{dW_i} = 0$$

for $dm_i = d\alpha$

$$\frac{dm_i}{dm_i} = 1 > 0;\ \frac{dw_i}{dm_i} = \frac{dP_y}{dm_i} = 0 \Rightarrow \frac{dh_i^P}{dm_i} > 0$$

for $dP_y = d\alpha$

$$\frac{dP_y}{dP_y} = 1 > 0;\ \frac{dw_i}{dP_y} = \frac{dm_i}{dP_y} = 0 \Rightarrow \frac{dh_i^P}{dP_y} > 0$$

To derive (3.11), (3.9*) is totally differentiated with respect to α. This yields:

$$\frac{dP_y}{d\alpha}F_i^h + \left[F_i^{hh}\frac{dh_i^y}{d\alpha} + F_i^{hm}\frac{dm_i}{d\alpha}\right]P_y = A\frac{dW_i}{d\alpha} + B\frac{dh_i^\ell}{d\alpha}$$

where A and B are defined as before. Except for an exogeneous change in the minimum labor constraint (where $d = d\bar{h}_i^x$), $(dh_i^\ell/d\alpha) = -(dh_i^y/d\alpha)$. Making this substitution and solving for $dh_i^y/d\alpha$:

$$[B + P_yF_i^{hh}]\frac{dh_i^y}{d\alpha} = A\frac{dW_i}{d\alpha} - \left[F_i^h\frac{dP_y}{d\alpha} + P_yF_i^{hm}\frac{dm_i}{d\alpha}\right]$$

Or:

$$\frac{dh_i^y}{d\alpha} = \frac{A}{(B+P_yF_i^{hh})}\frac{dW_i}{d\alpha} - \frac{1}{(B+P_yF_i^{hh}}\left[F_i^h\frac{dP_y}{d\alpha} + P_yF_i^{hm}\frac{dm_i}{d\alpha}\right]$$

where: $A > 0$, $(B + P_yF_i^{hh}) < 0$, $F_i^h > 0$ and $F_i^{hm} > 0$. Substituting the specific exogeneous variables for α now yields the following results:

for $dw_i = d\alpha$:

$$\frac{dW_i}{dw_i} > 0;\ \frac{dP_y}{dw_i} = \frac{dm_i}{dw_i} = 0 \quad \Rightarrow \quad \frac{dh_i^y}{dw_i} < 0$$

for $dW_i' = d\alpha$:

$$\frac{dW_i}{dW_i'} > 0;\ \frac{dP_y}{dW_i'} = \frac{dm_i}{dW_i'} = 0 \quad \Rightarrow \quad \frac{dh_i^y}{dW_i'} < 0$$

for $dm_i = d\alpha$

$$\frac{dW_i}{dm_i} > 0;\ \frac{dP_y}{dm_i} = 0;\ \frac{dm_i}{dm_i} = 1 > 0 \quad \Rightarrow \quad \frac{dh_i^y}{dm_i} \gtrless 0$$

for $dP_y = d\alpha$:

$$\frac{dW_i}{dP_y} > 0;\ \frac{dP_y}{dP_y} = 1 > 0;\ \frac{dm_i}{dP_y} = 0 \quad \Rightarrow \quad \frac{dh_i^y}{dP_y} \gtrless 0$$

This analysis changes somewhat if the minimum labor constraint changes. In this case:

$$-\frac{dh_i}{d\bar{h}_i^x} = \frac{d\bar{h}_i^x}{d\bar{h}_i^x} + \frac{dh_i^y}{d\bar{h}_i^x} = 1 + \frac{dh_i^y}{d\bar{h}_i^x}$$

Substituting this equality into the first equation on p. 122 and moving all of the expressions containing $dh_i^y/d\bar{h}_i^x$ on the left-hand side and all of the remaining terms to the right-hand side of the equation yields:

$$(P_y F_i^{hh} + B)\frac{dh_i^y}{d\bar{h}_i^x} = A\frac{dW_i}{d\bar{h}_i^x} - (F_i^h \frac{dP_y}{d\bar{h}_i^x} + F_i^{hm} \frac{dm_i}{d\bar{h}_i^x}) - B$$

where $(dP_y/d\bar{h}_i^x) = (dm_i/d\bar{h}_i^x = 0$ and $(dW_i/d\bar{h}_i^x) =$ $(w_i + P_y F_i^h (dh_i^y/d\bar{h}_i^x))$. Substituting these expressions into the above equation and solving for $dh_i^y/d\bar{h}_i^x$ now yields:

$$(P_y F_i^{hh} + B - AP_y F_i^h)\frac{dh_i^y}{dP_y} = Aw_i - B$$

Or:

$$\frac{dh_i^y}{d\bar{h}_i^x} = \frac{A}{(P_y F_i^{hh} + B - AP_y F_i^h)} w_i - \frac{B}{(P_y F_i^{hh} + B - AP_y F_i^h)} < 0$$

Bibliography

Batra, Raveendra N. "Technological Change in the Soviet Collective Farm." American Economic Review 64 (September 1974):594-603.

Clayton, Elizabeth Jean Myers and Oi, Walter Y. "A Peasant's View of a Soviet Collective Farm." American Economic Review 58 (March 1968):594-603.

Domar, Evsey. "The Soviet Collective Farm as a Producer Cooperative." American Economic Review 56 (September 1966):734-757.

Ward, Benjamin. "The Farm in Illyria:Market Syndicalism." American Economic Review 48 (September 1958):566-589.